GEOS 602 – Geophysical Fields
Jeff Freymueller (jeff.freymueller@gi.alaska.edu) 413B Elvey x7286

Problem Set #6 Due Tue March 23

Make sure you have read the rest of chapter 5 in Turcotte and Schubert.

1. Do Problem 5-23 in Turcotte and Schubert.

2. For a point mass buried at a depth z' (z positive downward), observed on an observation plane $z = z_0$, $z' > z_0$, the Fourier transform of the vertical acceleration of gravity for a point mass of anomalous mass ΔM is
$G_z(k) = \mathrm{FT}[g_z] = 2\pi G\Delta M e^{|k|(z_0 - z')}$
Now consider an arbitrary mass distribution.

   a. Show that the value of $\mathrm{FT}[g_z]$ at $k = 0$ is proportional to the total mass of the distribution.

   b. More generally, show that the "amplitude spectrum" satisfies the inequality
   $|FT[g_z]| \le 2\pi G\Delta M e^{-|k|z_1}$
   where ΔM is the total anomalous mass and z_1 is the depth to the top of the source distribution.

3. The *deflection of the vertical* is a measure of the local geoid slope. It is the angular distance between the local vertical as measured by a plumb bob (or spirit level) and the local vertical predicted by the reference ellipsoid (best fit to the geoid). If the geoid anomaly is $\Delta N(x)$, where ΔN varies only in the x-direction, the deflection of the vertical θ is
$$\theta = \frac{d\Delta N}{dx}$$
Isostasy was discovered as a result of geodetic measurements made in mountainous areas, specifically in the northern plains of India and the Himalaya. We will look at a simplification of this problem. Supposed that there is a linear mountain range with topography $h = h_0 \sin(2\pi x/\lambda)$.

   a. If the range is completely compensated by Airy compensation, what is the geoid anomaly ΔN?

   b. You make vertical angle measurements to measure the height of the highest peaks in the range. Suppose your observation point is at $x = \lambda/12$ and the peak is at elevation h_0, located at $x = \lambda/4$. Your vertical angle measurement will be biased by the deflection of the vertical. What is the error in elevation that results from this bias in your measurement for $\rho_c = 2600$ kgm^{-3}, $\rho_m = 3300$ kgm^{-3}, $h_0 = 6000$m, $\lambda =$ 200 km, and normal crustal thickness $H = 35$ km?

HINTS: 1. Assume that the mountain range is small enough that its gravity effect is not incorporated in the nominal reference gravity field, and that the reference geoid is horizontal here. Thus, if the angle between horizontal (reference ellipsoid) and the peak is α, the observed angle will be $\alpha - \theta$, because the telescope used for sighting will be tilted by an angle θ. 2. Read carefully sections 5-12 and 5-13. Assume the normal crustal thickness is H, and densities for the crust and mantle are ρ_c and ρ_m, respectively.